

Determination of the Apparent Viscosity of Dense Gas-Solids Emulsion by Magnetic Particle Tracking

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Abstract

When designing fluidised bed units a key to ensure efficient conversion is proper control of the mixing of the fuel in both lateral and axial directions in the bed. In order to mechanistically describe the mixing of fuel particles in a fluidised bed, there is a need to determine the apparent viscosity of the gas-solids emulsion, which determines the drag on the fuel particles.

In this work the apparent viscosity of a bed of spherical glass beads and air at minimum fluidisation was determined by means of the falling sphere method. Hereto the drag of the bed on a single immersed object was obtained by measuring the velocity of a negatively buoyant tracer with magnetic particle tracking (MPT). MPT allows for highly temporally and spatially resolved trajectories (10^{-3} s and 10^{-3} m, respectively) in all 3-dimensions. The bed consisted of glass beads with a narrow size distribution (215 to 250 μ m) and tracers with a size from 5 to 20 mm and densities from 4340 to 7500 kg/m³ were used. Hence, the literature, which typically covers data for velocities lying within or just above the Stoke flow regime ($0.002 < Re < 2.0$) could be expanded to Re numbers (53 to 152) well within the transition flow regime. The drag and apparent viscosity was compared to different fluid models and agreed well with the Newtonian model, when taking into account possible effects of the bed walls. Comparing the drag coefficient of data of free falling spheres and data of spheres falling with controlled velocities, the latter showed a dependence on the product of tracer diameter and falling velocity, $d_p u_t$, while the former was constant over $d_p u_t$. This indicates the method with controlled falling velocities to be intrusive and influencing the result of the apparent viscosity of the bed. Using the free falling sphere method this work obtained an apparent viscosity of 0.24 Pa s, which is consistent with values found in earlier literature for an emulsion of air and sand of similar size and density.

Keywords: drag coefficient; apparent viscosity; falling sphere method; magnetic particle tracking

1 Introduction

Despite being a widely applied technology for combustion and gasification of solid fuels, the design of fluidised bed (FB) units remains challenging. An important design requirement is to control the mixing of the fuel particles in both axial and lateral directions of the bed in order to ensure efficient conversion. Axial segregation may result in fuel particles sinking to the bottom of the bed, accumulating in poorly fluidised regions. On the other hand, lighter fuels may float and accumulate on the dense bed surface experiencing a reduced mass and heat transfer from the bed and with this a reduction of the reaction rate. Further, floating fuel particles will be more exposed to bubble eruptions, which enhance lateral mixing. Also, large furnace cross sections may result in lateral maldistribution of the fuel and, thereby in combustion intensity. Indirect gasifiers in a dual-bed system require control of the fuel transportation through the gasifier bed.

The dense bed in the bottom of a fluidised bed is classically described through two different phases, namely an emulsion phase consisting of the solids and gas at minimum fluidisation conditions and a gas phase consisting of rising bubbles, corresponding to a complex multiphase flow system. As indicated by the name, the emulsion phase possesses properties similar to a fluid, such as *apparent* density, viscosity and hydrostatic pressure. The forces which originate from these properties induce the mixing of the bed material and the fuel particles. While density and hydrostatic pressure of the emulsion phase of the bed can be quantified with pressure measurements, the apparent viscosity is

difficult to determine both theoretically and experimentally.

Not all methods of measuring viscosity in real fluids can be applied to gas-solids emulsions. Schügerl et al. [1] recommend the use of Couette viscometers, while Grace [2] determined the viscosity with X-radiography from the shape of bubbles rising in the bed. A common method used for all kind of fluids is the falling sphere method based on the Stokes' law, where a sphere of known diameter and density is falling freely through the fluid until settling at a constant velocity, the terminal velocity. In fluidised beds Daniels [3] obtained the drag coefficient using both free falling spheres in sand and ilmenite beds and spheres sinking with velocities controlled by a pulley system through a bed with spherical glass beads of different particle size [4, 5]. Rees et al. [6] used buoyant spheres, which were rising through beds of silica sand. For all measurements mentioned above the beds were at minimum fluidisation. In a later work Rees et al. [7] reviewed several techniques to measure viscosity in fluidised beds; including bubble shape, free falling sphere and Couette viscometer; and found good comparability between the methods, although the review did not include measurement by a pulley system as applied by Daniels [4, 5].

As many as there are measurement methods [7], there are contradicting theories of the rheology of fluidised beds. After evaluating different bed materials, tracer sizes and sinking velocities Daniels [4, 5] found a certain inconsistency in the results for the apparent bed viscosity, although Daniels was able to relate the drag coefficient to a function of the Reynolds number, the Froude number and the diameter ratio between the tracer and bed solids, valid across the range investigated. Wei and Chen [8] used Daniels [4, 5] measurement data complemented with data from their own falling sphere experiments. They state that the fluidised bed emulsion behaves like a Bingham fluid and calculated plastics viscosities and yield stresses for different sizes of glass beads. Unfortunately, little information regarding experimental and theoretical methods of both sources is available, which hampers the comparison to their findings. In contrast to Wei and Chen [8], Rees et al. [7] found the emulsion viscosity to be independent of the rate of shear. Thus, based on the theory of Newtonian fluids and Stokes' law, they used Daniels [3] measurement data and calculated the apparent viscosity by introducing a *defluidised* hood above the tracer resulting in a deviation from Stokes' law for non-compressible fluids. Their model results compare well with data found in literature showing that the emulsion viscosity increases with mean particle size of the bed material.

The aim of the present work is to expand the existing literature by determining the drag force of a fluidised bed of spherical glass beads on a single immersed tracer with high measurement accuracy. This is done by means of the falling sphere method, measuring the velocity of a negatively buoyant tracer, covering tracer size of 5 to 20 mm, with magnetic particle tracking (MPT) in a bed at minimum fluidisation. The MPT method was recently shown by the authors [9] to allow for the measuring of highly temporally and spatially resolved trajectories (~ 100 Hz, 1 mm) in all 3-dimensions. Hence, the tracer velocity is obtained at a much higher accuracy than data available in previous literature. Further, velocities in lateral directions are monitored, thus, measurements for which the sphere moved in the lateral direction can be disregarded. The work applies different tracer sizes and densities covering the interval used by previous works [3-6, 8], but also bigger tracers, hence, covering a range of much higher Re numbers than in previous work, which were typically limited within the Stokes' flow regime or slightly above. The Re numbers obtained in the present work range from 53 to 152, lying within the transition flow regime. The data is compared to different fluid models.

2 Method

2.1 Experimental setup

Experiments are carried out in a laboratory scale fluidised bed with a cross-sectional area of $0.17\text{m} \times 0.17\text{m}$, in which glass beads with a mean diameter of $230\text{ }\mu\text{m}$ were fluidised with air at room temperature. The fluidisation gas passes a porous plate with high pressure drop before entering the bed ($\Delta P_{\text{bed}}/\Delta P_{\text{distr}} \sim 0.12$). Table 1 gives an overview of the experimental parameters. Spherical tracers of four different diameters (5 to 20 mm) and with different densities (4340 to 7500 kg/m^3) as given Table 2 in were used.

Table 1. Physical parameters of the measurement setup

Parameter	Unit	
Bed dimension	m × m	0.17 × 0.17
Bed material density	kg/m ³	2600
Bed material size	µm	212 – 250
Bed height	m	0.13 – 0.16
Min. fluid. velocity	m/s	0.048

Table 2. Overview of tracers used

Density (kg/m ³)	Tracer 20 mm	Tracer 10 mm	Tracer 8 mm	Tracer 5 mm
X ₁	7500	7500	7500	7500
X ₂	6140	-	-	-
X ₃	4800	-	-	-
X ₄	4590	4340	-	-

As stated by Rees et al. [7], one of the difficulties of the falling sphere method in fluidised beds is the need of measuring in absence of bubbles, hence at minimum fluidisation velocity, u_{mf} , while maintaining a homogeneous fluidisation in the entire bed. In reality there are always poorly fluidised regions (partly due to segregation of the bed solids) and at the same time bubbles are formed in some locations due to local disturbances, e.g. by the tracer particle. To reduce these unwanted effects the bed material was sieved in order to narrow its particle size distribution to sizes between 212 and 250 µm. Additionally, prior to each measurement the bed is fluidised well (at around $2u_{mf}$). After the bed is sufficiently mixed, the gas velocity is reduced to u_{mf} and no bubbles are visible anymore, after which the tracer is immediately dropped into the bed. This is crucial in order to achieve u_{mf} conditions in the middle of the bed, where the tracer is dropped.

The MPT method for measuring the trajectories of the tracer was previously shown by the authors [9] to be suitable for application in down-scaled fluidised beds obtaining a high temporal and spatial resolution. In this work MPT is applied with a measurement frequency of 100 Hz, which is sufficiently high for the tracer velocities (up to 1 m/s) expected in the setup [9]. MPT allows for tracking the tracer in all three dimensions. The velocity of the tracer is calculated as a moving average over eight consecutive measurement points. Before extracting the velocity data, the trajectories of each measurement are checked as only data series from free falling tracers with little displacement in lateral direction are suitable. Figure 1 shows the typical velocity trajectories over bed height for three different tracers. For each tracer the measured maximum and minimum falling velocity are displayed. Each tracer is released on the bed surface (marked in the figure as 'Start') and accelerates until yielding a reasonably constant falling velocity in the middle of the bed (assumed to be the terminal velocity) and finally decelerating to end up on the vicinity of the gas distributor. After up to 10 repetitions for each tracer a mean terminal velocity is extracted from all measurements.

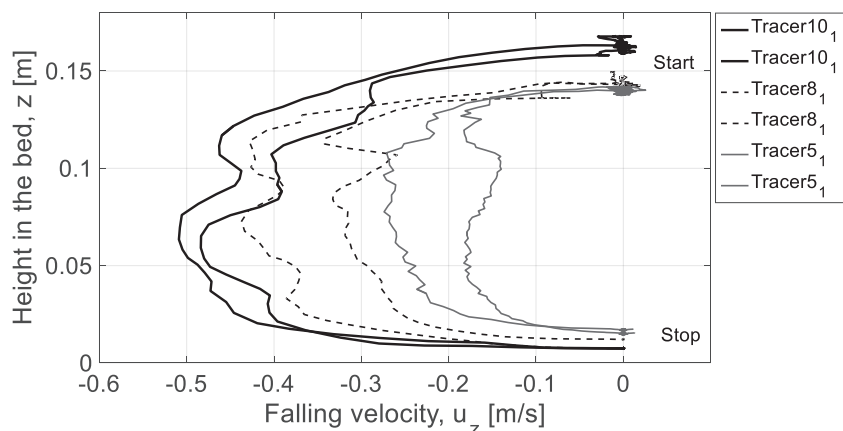


Figure 1. Maximum and minimum measured falling velocity, u_z , over bed height, z , for three different tracers, Tracer 10₁, 8₁ and 5₁, after 10 repetitions.

2.2 Theory

In Newton's equation of motion the forces acting on a sphere immersed in a fluidised bed can be identified as gravitational, buoyancy and drag force. When reaching the terminal velocity, the acceleration of the sphere is zero. At minimum fluidisation the velocity of the emulsion is assumed to be close to zero and the force balance equation can be solved for the terminal velocity as:

$$u_t = \left(\frac{4gd_p(\rho_p - \rho_e)k_w}{3C_D\rho_e} \right)^{0.5} \quad (1)$$

where d_p , ρ_p and u_p are the diameter, density and velocity of the tracer particle, ρ_e is the density of the emulsion, which can be determined by pressure measurement, and C_D is the drag coefficient and k_w is a correction factor for the presence of the wall of the bed included by Rees et al. [7] included, based on a definition by Perry and Green [10] where

$$k_w = 1.427\beta^2 - 2.374\beta + 1.005 \quad (2)$$

for flows with $Re < 100$, with β being the ratio of the particle diameter to the bed diameter. Within $100 < Re < 10\,000$ the parameter k_w is given by:

$$k_w = \frac{1 - \beta^2}{\sqrt{1 + \beta^4}} \quad (3)$$

Using Eq. (1) the drag coefficient can be directly calculated from the in-bed terminal velocity ("free in-bed falling velocity") obtained in the experiments.

If the flow regime is known the Reynolds number can be calculated from the drag coefficient [11]. For low Reynolds numbers ($Re < 0.5$) the drag coefficient can be calculated with the analytical solution proposed by Stokes [12]. However, for $Re > 0.5$ the flow might separate from the tracer particle and for this transition regime a common correlation used for spherical particles is the one by Schiller and Naumann [13]:

$$C_D = \frac{24}{Re} (1 + 0.15Re^{0.687}) \quad (4)$$

Having this, the apparent viscosity of the bed emulsion, μ_e , is given by the definition of the Reynolds number, which reads:

$$Re = \frac{d_p \rho_e u_t}{\mu_e} \quad (5)$$

Table 3 compares the experimental conditions in this work with those of investigations available in literature.

Table 3. Results from this work and corresponding data available in literature

Source	Bed material	Size (μm)	Bulk density at u_{mf} (kg/m^3)	Tracer size (mm)	Tracer density (kg/m^3)
This work	Glass beads	212-250	1590	5; 8; 10; 20	7500; 6140; 4800; 4600; 4340
Daniels 1959	Sand, ilmenite	200-300	1400; 2350	2; 2.7; 3.1; 3.9; 4.7; 5.1; 6.3	16 600; 10 200; 7800; 6400; 4500
Daniels 1962	Glass beads	65-100; 100-135; 135-170; 170-200	1320; 1380	9.53; 6.35; 5.54; 4.75	n.a.
Daniels 1965	Glass beads	65-100; 100-135; 135-170; 170-200	1320; 1380	9.53; 6.35; 5.54; 4.75	n.a.
Rees 2005	Sand	212-300	1345	9.0; 13.2	905; 1060; 1210; 1320

3 Results and Discussion

3.1 Drag coefficient

Daniels [4] measured different terminal velocities of a falling sphere balanced with a pulley system, with which different falling velocities could be produced. The work investigated how the drag coefficient changes over the product of the particle diameter and the terminal velocity. Figure 2 plots the drag coefficient, C_D , against the product of the particle diameter and the terminal velocity, $d_p u_t$, and shows how data from Daniels [4, 5] compares to this work and Daniels earlier work [3], the latter two obtained with free falling tracers. As for the values of the drag coefficient, C_D , data in Daniels [3] was recalculated with eq. (1), while for the experiments in Daniels [4, 5] they were taken as listed in [5].

As can be seen in the figure, with the pulley system deployed by Daniels [4], it was possible to produce drag coefficients over a wider range than what was possible in the free falling systems applied by Daniels [3] and this work, are nearly constant across the range of the product $d_p u_t$. This is questioning the comparability of both methods, hence, indicating the measurement method with the pulley system to be intrusive and influencing its results. In fact, the difference in the drag coefficients resulting from the two methods were mentioned earlier [4].

Comparing the results of both free falling sphere experiments, the lower drag coefficient in the present work can be explained by the larger tracer size and the different bed material: although density and mean size of the bed solids are similar, Daniels [3] used a wider particle size distribution and sand particles with a lower sphericity (and thus higher friction) than the glass beads used in the present work. The glass beads used in this work are believed to produce less friction than the sand particles used in [3] due to their spherical shape. Typical values for the sphericity of sand can be found in Kunii and Levenspiel to be between 0.67 and 0.86 [14]. The sphericity of glass beads is assumed to be around 1. This as well as the bigger tracers used in the present work result in higher velocities, hence, in a higher product of particle diameter and terminal velocity.

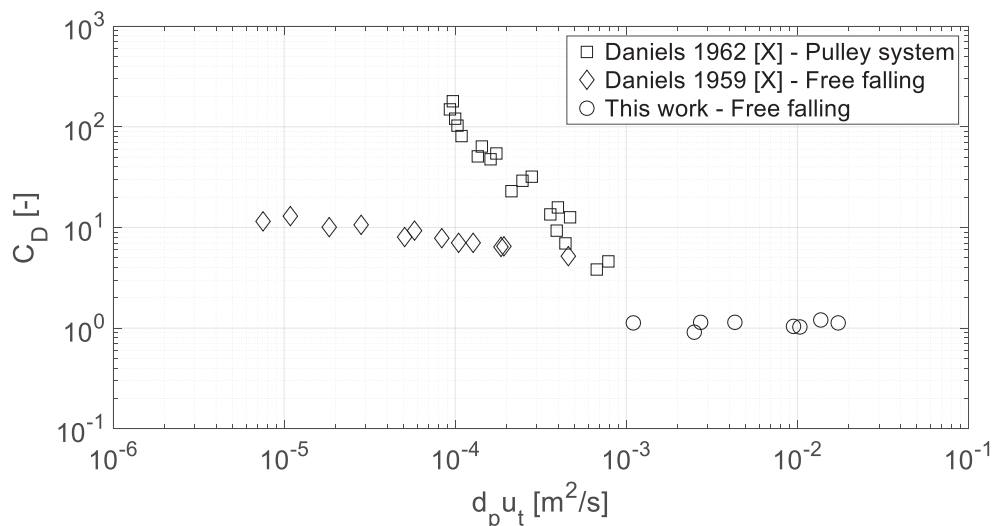


Figure 2. Drag coefficient, C_D , over the product of particle diameter, d_p , and terminal velocity, u_t , as investigated by Daniels [4], comparing data from Daniels [3, 4] with this work.

3.2 Fluid model

Figure 3 plot the apparent viscosity of the bed emulsion against the ratio $d_p/3u_t$, comparing data from (a) Daniels [4, 5] with (b) this work and Daniels [3]. Here, the apparent viscosity was calculated as done by Wei and Chen [8], assuming the bed emulsion to behave like a Bingham plastic, i.e. the viscosity consists of an offset value and a term linearly increasing with shear stress, thus modified the relationship of the Reynolds number and the bed viscosity to be:

$$Re_m = \frac{d_p \rho_e u_t}{\mu_e}, \quad \mu_e = \mu + \frac{\tau d_p}{3u_t}, \quad (6)$$

where μ represents the plastic viscosity and τ the yield stress of the bed material. Wei and Chen found the values for μ and τ by plotting Daniels [4] apparent viscosity against the ratio of particle diameter and terminal velocity. As can be seen in the figures the trends for free falling spheres are opposite to the trend found by Wei and Chen [8] for data obtained through a pulley system. While apparent viscosity for the latter is linearly increasing with $d_p/3u_t$, it shows a decreasing trend in the case of free falling experiments. Thus, the Bingham model is not suitable as general description of the apparent viscosity of gas-solids emulsions.

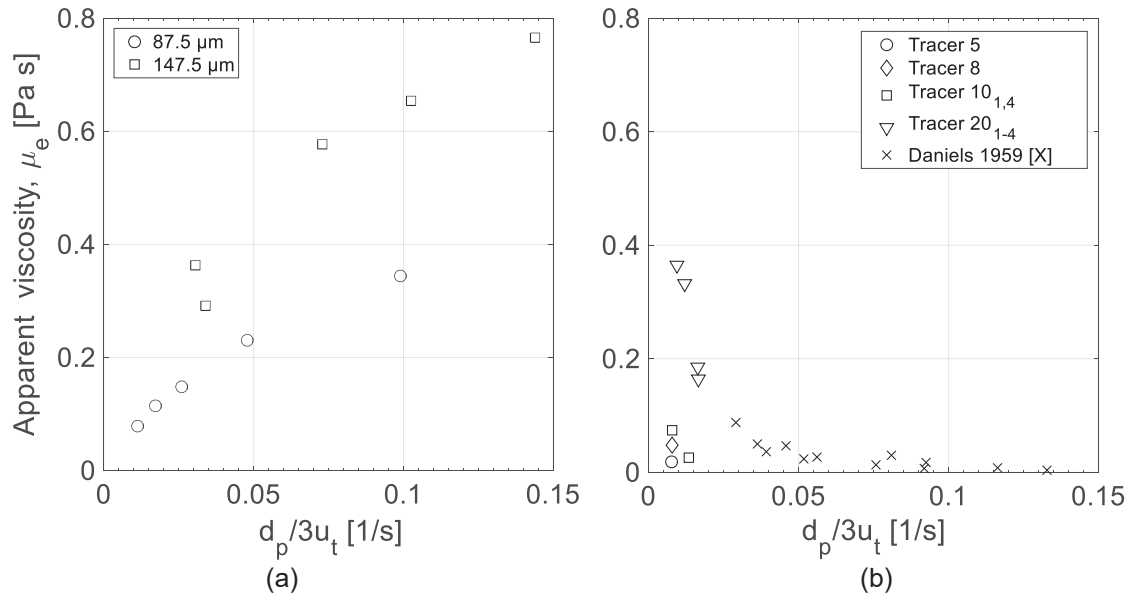


Figure 3. Apparent viscosity, μ_e , over the ratio $d_p/3u_t$ using Bingham plastic model [8]. Comparing data from (a) Daniels [5] with (b) this work and Daniels [3].

Rees et al. [6, 7] calculated the apparent viscosity by using the Newton fluid model, but introducing a defluidised zone above the tracer, a so called *defluidised hood*, which may act on the tracer, thus influencing its falling velocity. It is not obvious if for a negatively buoyant particle (tracer and *defluidised hood*) the hood will stay attached to the tracer or if the tracer might fall faster and break away from the hood.

Figure 4 shows the apparent viscosity over the terminal velocity, comparing Daniels work [3] with data obtained by this work. For Daniels work [3] it was assumed by Rees et al. [7] that the hood was attached to the tracer, thus results were obtained by including the influence of the hood in the calculations resulting in a viscosity of around 0.5 Pa s (Δ). As can be seen in Figure 4 terminal velocities obtained by this work are much higher than the velocities of Daniels [3], which increases the probability of having a detached hood. For the present measurements, the apparent viscosity is calculated both excluding the influence of a *defluidised hood* (eq. (1)) and with a hood attached, hence, using the equations presented by Rees et al. [6, 7]. The apparent viscosity varies between 0.19 and 0.29 Pa s for a detached hood (which is expected to be the case, given the large terminal velocities covered by experiments in this work) and between 0.036 and 0.064 Pa s when accounting for the de-fluidised hood. The values are below the viscosities from Daniels [3] measurements, which as mentioned above is explained by the lower friction of spherical glass beads in comparison to sand.

Rees et al. [7] evaluated different methods to measure the viscosity in fluidised beds found in literature and showed for increasing diameter of bed solids how the viscosity increases. Comparing the viscosity resulting from the different measurement methods with magnitudes summarised in their work the higher viscosity (0.19 – 0.29 Pa s) found for the material presented here is reasonable for glass beads of the size used. Further, as mentioned before the velocities seem to be too high for the

hood to stay attached to the tracer or to reform quick enough to influence the velocity measurements. It is therefore assumed the tracers fall freely without *defluidised hood* and an apparent viscosity of the bed emulsion of around 0.24 Pa s is obtained.

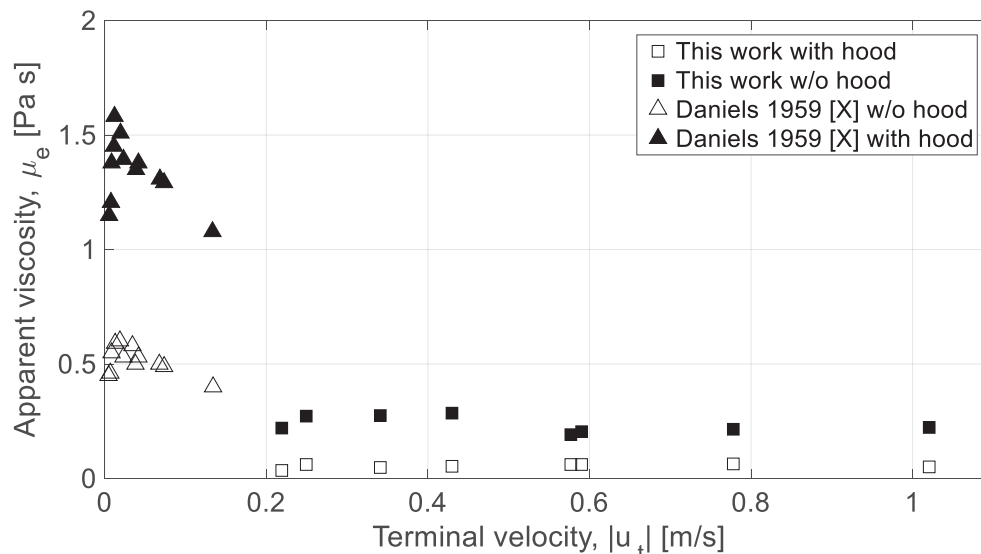


Figure 4. Apparent viscosity, μ_e , calculated as proposed by Rees et al. [7] over the absolute terminal velocity, $|u_t|$, comparing data from this work and Daniels [3].

4 Conclusions

The apparent viscosity of the bed emulsion of spherical glass beads with a particle diameter of 215 to 250 μm and air was studied with the falling sphere method, while maintaining the bed at minimum fluidisation exhibiting a Newtonian character.

The drag force was obtained by measuring the falling velocity of a spherical tracer with magnetic particle tracking and was compared to values found in the literature, both from measurements of free falling spheres and of spheres with controlled velocities [3, 4]. The comparison indicates the measurement method of spheres with controlled velocities to be intrusive, hence influencing the drag of the emulsion on the tracer and therefore the result of the apparent viscosity.

Although the data presented in this work cover a range of Re numbers (53 to 152) in the transition flow regime, the work results in a bed viscosity of around 0.24 Pa s showing good agreement with numbers found in literature [7] for the particle size of the bed material used.

5 References

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